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Applications on Generalized Two-Dimensional Fractional Cosine Transform

V. D. Sharma, S. A. Khapre
*Department of Mathematics Arts, Commerce & Science College, Amravati (M.S.) 444606 India
**Department of Mathematics, P. R. Patil College of Engineering and Technology, Amravati.

Abstract*-Fourier transform can be generalized into the fractional Fourier transform (FRFT), Linear Canonical transform (LCT) and Simplified fractional Fourier transform. They extend the utilities of original Fourier transform and can solve many problems that can't solved well by original Fourier transform. in this paper fractional Cosine transform is extended in the distributional generalized sense and examples on fractional Cosine transform are discussed.

Keywords- **fractional Fourier transforms, fractional Cosine transform fractional Sine transform.**

I. INTRODUCTION

The theory of integral transform is presented a direct and systematic technique for the presentation of classical and distribution theory and it is one of the well-known technique used for function transformation [3]. For almost two centuries the method of function transformation has been successfully in solving many problems in engineering mathematical Physics and applied mathematics. Integral transform method proved to be of great importance in initial and boundary value problem of partial differential Equation [8]. Extension of some transformation to generalized function have been done from time to time and their properties have been studied by various mathematicians Zemanian [1] and Pathak [2] had studied number of transform as generalized functions. In the 80's a Generalized Fourier transform was introduced for signal processing, fifty year after its introduction in the field of pure mathematics. The fractional Fourier transform, which is generalization of the ordinary Fourier transform. The fractional Cosine and Sine transform closely related to the fractional Fourier transform [7]. As the generalization of fractional Cosine transform has been used in several area including optical analysis and signal processing. In our previous work [4, 5, 6] we had defined. Motivated by the above, in this paper we have studied two dimensional fractional Cosine transform in distributional sense and discuss examples on generalized two dimensional fractional Cosine transform.

A. Two dimensional generalized fractional Cosine transform

Two dimensional fractional Cosine transform with parameter α f(x, y) denoted by $F_C^\alpha(x,y)$ perform a linear operation given by the integral transform.

$$F_C^\alpha\{f(x,y)\}(u,v) = \int_0^\infty\int_0^\infty f(x,y)K_\alpha(x,y,u,v)\,dx\,dy \tag{1.1}$$

Where the kernel,

$$K_C^\alpha(x,y,u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy) \tag{1.2}$$

B. The test function space E

An infinitely differentiable complex valued function $\emptyset$ on R^n belongs to $E(R^n)$ if for each compact set $I\subset S_{a,b}$, where, $S_{a,b}=\{x,y:x,y\in R^n,|x|\le a,|y|\le b,a>0,b>0\}, I\in R^n$

$$\gamma_{E_{p,q}}(\emptyset)=\sup_{x,y}\left|D_{x,y}^{p,q}\emptyset(x,y)\right|<\infty$$

Where, p, q =1, 2, 3….

Thus $E(R^n)$ will denote the space of all $\emptyset\in E(R^n)$ with support contained in $S_{a,b}$ Note that space E is complete and therefore a Frechet space. Moreover, we say that f is a fractional Cosine transformable, if it is a member of E^*, the dual space of E.

II. DISTRIBUTIONAL TWO-DIMENSIONAL FRACTIONAL COSINE TRANSFORM

The two dimensional distributional fractional Cosine transform of $f(x,y)\in E^*(R^n)$ defined by

$$F_C^\alpha\{f(x,y)\}=F^\alpha(u,v)=\langle f(x,y),K_\alpha(x,y,u,v)\rangle \tag{2.1}$$

$$K_C^\alpha(x,y,u,v)=\sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy) \tag{2.2}$$

Where , RHS of equation (2.1) has a meaning as the application of $f\in E^*$ to $K_\alpha(x,y,u,v)\in E$

III. EXAMPLES

3.1 If $F_C^\alpha\{f(x,y)\}(u,v)$ denotes generalized two dimensional fractional Cosine transform of f(x,y)then **,**

$$F_C^\alpha\{1\}(u,v)\}=\frac{\sqrt{\pi}}{2\sqrt{2}}\sqrt{tan^2\theta-itan\theta}\,e^{-i/2(u^2+v^2)tan\theta}$$

Solution:

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$$F_C^\alpha\{1\}(u,v) = \int_0^\infty\int_0^\infty 1\sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy)\,dxdy$$

$$F_C^\alpha\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(u^2+v^2)cot\alpha}{2}}\int_0^\infty\int_0^\infty 1\,e^{\frac{i(x^2+y^2)cot\alpha}{2}}\cos(cosec\alpha.ux)\cos(cosec\alpha.vy)\,dxdy$$

Let, A= $\sqrt{\frac{1-icot\alpha}{2\pi}}$ **B=**$e^{\frac{i(u^2+v^2)cot\alpha}{2}}$

$$F_C^\alpha\{1\}(u,v) = AB\int_0^\infty e^{\frac{i(x^2)cot\alpha}{2}}\cos(cosec\alpha.ux)\,dx\int_0^\infty e^{\frac{i(y^2)cot\alpha}{2}}\cos(cosec\alpha.vy)\,dy$$

Let, a= $\frac{cot\alpha}{2}$ **, b=**$cosec\alpha.u$**, c=**$cosec\alpha.v$

$$F_C^\alpha\{1\}(u,v) = AB\int_0^\infty e^{ix^2a}\cos(bx)\,dx\int_0^\infty e^{iy^2a}\cos(cy)\,dy$$

$$F_C^\alpha\{1\}(u,v) =$$

$$AB\left[\left(\frac{-1}{4\sqrt{a}}(-1)^{\frac{3}{4}}\sqrt{\pi}e^{\frac{-ib^2}{4a}}\right)\left(\begin{matrix}erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax-b)\right)+\\(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax+b)\right)\end{matrix}\right)\right]_0^\infty\left[\left(\frac{-1}{4\sqrt{a}}(-1)^{\frac{3}{4}}\sqrt{\pi}e^{\frac{-ic^2}{4a}}\right)\left(\begin{matrix}erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax-c)\right)+\\(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax+c)\right)\end{matrix}\right)\right]_0^\infty$$

$$F_C^\alpha\{1\}(u,v) = AB\frac{-1}{4\sqrt{a}}(-1)^{\frac{3}{4}}\sqrt{\pi}\frac{-1}{4\sqrt{a}}(-1)^{\frac{3}{4}}\sqrt{\pi}\left(e^{\frac{-i(b^4+c^4)}{4a}}\right)$$

$$\left[\left(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax-b)\right)+(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax+b)\right)\right)\right]_0^\infty\left[\left(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax-c)\right)+(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax+c)\right)\right)\right]_0^\infty$$

$$F_C^\alpha\{1\}(u,v) = AB\frac{-1}{4\sqrt{a}}(-1)^{\frac{3}{4}}\sqrt{\pi}\frac{-1}{4\sqrt{a}}(-1)^{\frac{3}{4}}\sqrt{\pi}\left(e^{\frac{-i(b^2+c^2)}{4a}}\right)$$

$$\left[\left(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax-c)\right)+(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax+c)\right)\right)\right]_0^\infty$$

$$\left[\left(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax-c)\right)+(erfi\left(\frac{\sqrt[4]{-1}}{2\sqrt{a}}(2ax+c)\right)\right)\right]_0^\infty$$

$$F_C^\alpha\{1\}(u,v) = AB\frac{1}{16a}(-1)^{\frac{3}{2}}\pi\left(e^{\frac{-i(b^4+c^4)}{4a}}\right)$$

$$\left[\begin{matrix}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(\infty)\right)+(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(\infty)\right)\right)-\\\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(-b)\right)+(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(b)\right)\right)\end{matrix}\right]$$

$$\left[\begin{matrix}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(\infty)\right)+(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(\infty)\right)\right)-\\\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(-c)\right)+(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(c)\right)\right)\end{matrix}\right]$$

Here $erfi(\sqrt{i}\infty) = \sqrt{i}$

H

$$F_C^\alpha\{1\}(u,v) = AB\frac{1}{16a}(-1)^{\frac{3}{2}}\pi\left(e^{\frac{-i(b^2+c^2)}{4a}}\right)$$

$$\left[(\sqrt{i}+\sqrt{i}) - erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(b)\right)+(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(b)\right)\right]$$

$$\left[(\sqrt{i}+\sqrt{i})-\left(-erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(c)\right)+(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(c)\right)\right)\right]$$

$$F_C^\alpha\{1\}(u,v) = AB\frac{1}{16a}(-1)^{\frac{3}{2}}\pi\left(e^{\frac{-i(b^2+c^2)}{4a}}\right)[(\sqrt{i}+\sqrt{i})][(\sqrt{i}+\sqrt{i})]$$

$$F_C^\alpha\{1\}(u,v) = AB\frac{1}{16a}(-1)^{\frac{3}{2}}\pi\left(e^{\frac{-i(b^2+c^2)}{4a}}\right)[2\sqrt{i}][2\sqrt{i}]$$

$$F_C^\alpha\{1\}(u,v) = AB\frac{1}{16a}(-1)^{\frac{3}{2}}\pi\left(e^{\frac{-i(b^2+c^2)}{4a}}\right)4i$$

$$F_C^\alpha\{1\}(u,v) = AB\frac{(-1)^{\frac{3}{2}}\pi}{4a}\left(e^{\frac{-i(b^2+c^2)}{4a}}\right)i$$

$$F_C^\alpha\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(u^2+v^2)cot\alpha}{2}}\frac{i(-1)^{\frac{3}{2}}\pi}{4a}\left(e^{\frac{-i(b^2+c^2)}{4a}}\right)$$

$$F_C^\alpha\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(u^2+v^2)cot\alpha}{2}}\frac{i(-1)^{\frac{3}{2}}\pi}{4a}\left(e^{\frac{-i(csc^2\alpha.u^2+csc^2\alpha.v^2)}{4\frac{cot\alpha}{2}}}\right)$$

$$F_C^\alpha\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(u^2+v^2)cot\alpha}{2}+\frac{-i(csc^2\alpha.u^2+csc^2\alpha.v^2)}{2cot\alpha}}\frac{i(-1)^{\frac{3}{2}}\pi}{4a}$$

$$F_C^\alpha\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i}{2}\left(\left(\frac{cot^2\alpha-csc^2\alpha}{cot\alpha}\right)u^2+\left(\frac{cot^2\alpha-csc^2\alpha}{cot\alpha}\right)v^2\right)}\frac{i(-1)^{\frac{3}{2}}\pi}{4a}$$

$$F_C^\alpha\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i}{2}(u^2+v^2)\left(\frac{cot^2\alpha-csc^2\alpha}{cot\alpha}\right)}\frac{i(-1)^{\frac{3}{2}}\pi}{4a}$$

$$F_C^\alpha\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{-i}{2}(u^2+v^2)\frac{1}{cot\alpha}}\frac{i(-1)^{\frac{3}{2}}\pi}{4a}$$

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$$F_C^{\alpha}\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{-i}{2}(u^2+v^2)tan\alpha}\frac{i(-1)^{\frac{3}{2}}\pi}{4a}$$

$$F_C^{\alpha}\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{-i}{2}(u^2+v^2)tan\alpha}\frac{i^4\pi}{4a}$$

$$F_C^{\alpha}\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{-i}{2}(u^2+v^2)tan\alpha}\frac{i^4\pi}{4\frac{cot\alpha}{2}}$$

$$F_C^{\alpha}\{1\}(u,v) = \sqrt{\frac{1-icot\alpha}{8\pi}}\, e^{\frac{-i}{2}(u^2+v^2)tan\alpha}\frac{\pi}{cot\alpha}$$

$$F_C^{\alpha}\{1\}(u,v) = \sqrt{\frac{(1-icot\alpha)cot\alpha\pi^2}{8\pi}}\, e^{\frac{-i}{2}(u^2+v^2)tan\alpha}$$

$$F_C^{\alpha}\{1\}(u,v) = \frac{\sqrt{\pi}}{2\sqrt{2}}\sqrt{tan^2\alpha - itan\alpha}\, e^{\frac{-i}{2}(u^2+v^2)tan\alpha}$$

3.2. If $F_c^{\alpha}\{f(x,y)\}(u,v)$ denotes generalized two dimensional fractional Cosine transform of f(x, y) then $F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = K_C^{\alpha}(a,b,u,v)$

Solution:

$$F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = \int_0^{\infty}\int_0^{\infty}(\delta(x-a).\delta(y-b))$$

$$\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\cos(cosec\alpha.ux)\,dy$$

$$.\cos(cosec\alpha.vy)\,dx$$

$$F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(u^2+v^2)cot\alpha}{2}}$$

$$\int_0^{\infty}\int_0^{\infty}(\delta(x-a).\delta(y-b))\, e^{\frac{i(x^2+y^2)cot\alpha}{2}}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy)\,dx\,dy$$

LLet A=$\sqrt{\frac{1-icot\alpha}{2\pi}}$ $\boldsymbol{B} = e^{\frac{i(u^2+v^2)cot\alpha}{2}}$

$$F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = AB\int_0^{\infty}\delta(x-a)\, e^{\frac{i(x^2)cot\alpha}{2}}\cos(cosec\alpha.ux)\,dx$$

$$\int_0^{\infty}\delta(y-b)\, e^{\frac{i(y^2)cot\alpha}{2}}\cos(cosec\alpha.vy)\,dy$$

$$F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = AB\, e^{\frac{i(a^2)cot\alpha}{2}}\cos(cosec\alpha.ua)$$

$$.\, e^{\frac{i(b^2)cot\alpha}{2}}\cos(cosec\alpha.vb)$$

We know that $\int_0^{\infty}\delta(t-a)\,\varphi(t)\,dt = \varphi(a)$

$$F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = ABe^{\frac{i(a^2+b^2)cot\alpha}{2}}\cos(cosec\alpha.ua).\cos(cosec\alpha.vb)$$

$$F_C^{\alpha}\{(\delta(x-a).\delta(y-b))\} = K_C^{\alpha}(a,b,u,v)$$

3.3. If $F_c^{\alpha}\{f(x,y)\}(u,v)$ denotes generalized two dimensional fractional Cosine transform of f(x,y) then

$$F_C^{\alpha}\{cosx.cosy\} = \sqrt{\frac{-\pi(1-icot\alpha)}{8cot^2\alpha}}\, e^{\frac{i}{2}((u^2+v^2)cot\alpha - tan\alpha(csc^2\alpha u^2+csc^2\alpha v^2+2)}$$

$$\cos(sec\alpha.u).\cos(sec\alpha.v)$$

Solution:

$$F_C^{\alpha}\{cosx.cosy\}(u,v) = \int_0^{\infty}\int_0^{\infty}cosx.cosy\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy)\,d$$

$$F_C^{\alpha}\{cosx.cosy\}(u,v) = AB\int_0^{\infty}\int_0^{\infty}cosx.cosy$$

$$e^{\frac{i(x^2+y^2)cot\alpha}{2}}\cos(cosec\alpha.ux).\cos(cosec\alpha.vy)\,dx\,dy$$

Let A=$\sqrt{\frac{1-icot\alpha}{2\pi}}$, $\boldsymbol{B} = e^{\frac{i(u^2+v^2)cot\alpha}{2}}$

$$F_C^{\alpha}\{cosx.cosy\}(u,v) = AB\int_0^{\infty}\int_0^{\infty}e^{\frac{i(x^2+y^2)cot\alpha}{2}}\frac{1}{2}(\cos(csc.u+1)x +$$

$$\frac{1}{2}(\cos(csc.v+1)y + \cos(csc.v-1)y).dx\,dy$$

$$F_C^{\alpha}\{cosx.cosy\}(u,v) = AB\frac{1}{4}\int_0^{\infty}\int_0^{\infty}e^{\frac{i(x^2+y^2)cot\alpha}{2}}(\cos(csc.u+1)x +$$

$$(\cos(csc.v+1)y + \cos(csc.v-1)y).dx\,dy$$

Let $b_1 = (csc.u+1)$, $b_2 = (csc.u-1)$, $c_1 = (csc.v+1), c_2 = (csc.v-1)$,$\frac{cot\alpha}{2} = a$

$$F_C^{\alpha}\{cosx.cosy\}(u,v) = AB\frac{1}{4}\int_0^{\infty}e^{iax^2}(\cos b_1x + \cos b_2x)dx\int_0^{\infty}e^{iay^2}(\cos c_1y + \cos c_2y)\,dy$$

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$$F_C^{\alpha}\{cosx.cosy\}(u,v) = AB\,\frac{1}{64a}(-1)^{\frac{3}{2}}\pi \left[\begin{pmatrix} e^{\frac{-ib_1^2}{4a}}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ax-b_1)\right)+erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ax+b_1)\right)\right)+ \\ e^{\frac{-ib_2^2}{4a}}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ax-b_2)\right)+erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ax+b_2)\right)\right) \end{pmatrix} \begin{pmatrix} e^{\frac{-ic_1^2}{4a}}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ay-c_1)\right)+erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ay+c_1)\right)\right)+ \\ e^{\frac{-ic_2^2}{4a}}\left(erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ay-c_2)\right)+erfi\left(\frac{\sqrt{i}}{2\sqrt{a}}(2ay+c_2)\right)\right) \end{pmatrix}\right]_0^{\infty}$$

$$F_C^{\alpha}\{cosx.cosy\}(u,v) = AB\,\frac{i^3\pi}{64a}\left[\left(e^{\frac{-ib_1^2}{4a}}(2\sqrt{i})+e^{\frac{-ib_2^2}{4a}}(2\sqrt{i})\right)\left(e^{\frac{-ic_1^2}{4a}}(2\sqrt{i})+e^{\frac{-ic_2^2}{4a}}(2\sqrt{i})\right)\right]$$

$$F_C^{\alpha}\{cosx.cosy\}(u,v) = AB\,\frac{-i\pi}{16a}\left[\left(e^{\frac{-ib_1^2}{4a}}+e^{\frac{-ib_2^2}{4a}}\right)\left(e^{\frac{-ic_1^2}{4a}}+e^{\frac{-ic_2^2}{4a}}\right)\right]$$

$$F_C^{\alpha}\{cosx.cosy\}(u,v) = AB\,\frac{-i\pi}{8cot\alpha}\left[\left(e^{\frac{-i(csc.u+1)^2}{2cot\alpha}}+e^{\frac{-i(csc.u-1)^2}{2cot\alpha}}\right)\left(e^{\frac{-i(csc.v+1)^2}{2cot\alpha}}+e^{\frac{-i(csc.v-1)^2}{2cot\alpha}}\right)\right]$$

$$F_C^{\alpha}\{cosx.cosy\}(u,v) =$$
$$AB\,\frac{-i\pi}{8cot\alpha}\left[\begin{matrix}\left(e^{\frac{-itan\alpha}{2}(csc^2\alpha u^2+1+2csc\alpha u)}+e^{\frac{-itan\alpha}{2}(csc^2\alpha u^2+1-2csc\alpha u)}\right) \\ \left(e^{\frac{-itan\alpha}{2}(csc^2\alpha v^2+1+2csc\alpha v)}+e^{\frac{-itan\alpha}{2}(csc^2\alpha v^2+1-2csc\alpha v)}\right)\end{matrix}\right]$$

$$F_C^{\alpha}\{cosx.cosy\}(u,v) =$$
$$AB\,\frac{-i\pi}{8cot\alpha}e^{\frac{-itan\alpha}{2}(csc^2\alpha u^2+1)}e^{\frac{-itan\alpha}{2}(csc^2\alpha v^2+1)}$$
$$[(e^{isec\alpha u}+e^{-isec\alpha u})(e^{isec\alpha v}+e^{-isec\alpha v})]$$

$$F_C^{\alpha}\{cosx.cosy\}(u,v) = AB\,\frac{-i\pi}{8cot\alpha}e^{\frac{-itan\alpha}{2}(csc^2\alpha u^2+csc^2\alpha v^2+2)}$$
$$2\cos(sec\alpha.u).2\cos(sec\alpha.v)$$

$$\frac{-i\pi}{2cot\alpha}e^{\frac{-itan\alpha}{2}(csc^2\alpha u^2+csc^2\alpha v^2+2)}\cos(sec\alpha.u)$$
$$.\cos(sec\alpha.v)$$

$$F_C^{\alpha}\{cosx.cosy\}(u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(u^2+v^2)cot\alpha}{2}}$$

$$F_C^{\alpha}\{cosx.cosy\}(u,v) = \sqrt{\frac{(-i)^2(1-icot\alpha)\pi}{8cot^2\alpha}}\,e^{\frac{i}{2}((u^2+v^2)cot\alpha-tan\alpha(csc^2\alpha u^2+csc^2\alpha v^2+2)}\cos(sec\alpha.u).\cos(sec\alpha.v)$$

$$F_C^{\alpha}\{cosx.cosy\}(u,v) =$$
$$\sqrt{\frac{-\pi(1-icot\alpha)}{8cot^2\alpha}}\,e^{\frac{i}{2}((u^2+v^2)cot\alpha-tan\alpha(csc^2\alpha u^2+csc^2\alpha v^2+2)}$$
$$\cos(sec\alpha.u).\cos(sec\alpha.v)$$

IV. CONCLUSION

We have extended two-dimensional fractional Cosine transform in the distributional generalized sense and solved some examples of generalized two-dimensional fractional Cosine transform.

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AUTHOR’S PROFILE

Dr. V.D. Sharma, is an Assistant Professor in the department of mathematics, Arts, Commerce and Science College, Kiran Nagar, Amravati (M.S).,India. She has obtained her doctoral degree from SGB Amravati University Amravati. She has published more than 40 articles in the field of integral transforms. Six research students are working under her supervision

Ku.Shilpa A.Khapre, is an Assistant Professor in the Department Mathematics, P.R.Patil College Of Engineering and Technology ,Amravati. She has obtained Msc. ,M..Phil.(Math),from SGB Amravati University, Amravati., She has Registered Ph.D. Under the Guidance of Dr.V.D.Sharma. She has 4 research articles to her credit.